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PD-003-1163005

Seat No. ______

M. Sc. (Sem. III) (CBCS) Examination

June / July - 2018

EMT - 3011 : Mathematics

(Differential Geometry)

Faculty Code : 003
Subject Code : 1163005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are 5 questions.
(2) Attempt all the questions.
(3) Figures to the right indicate full marks.

1 Attempt any **seven** : **14**

(1) Define : Functions of class k.
(2) Define : Right circular helix.
(3) Define : Reparametrization of a curve.
(4) What is regular curve segment ?
(5) Define : Length of a regular curve segment.
(6) Define with example : An open subset of R^2.
(7) Define : Simple surface.
(8) Define : The tangent plane and the normal plane.
(9) Define : Normal curvature and Geodesic curvature.
(10) Define : Unit speed curve.

2 Attempt the following questions : **14**

(a) Show that the curve $\alpha(t) = (\sin 3t \cos t, \sin 3t \sin t, 0)$ is regular. Also find the equation of tangent line to α at $t = \frac{\pi}{3}$.

OR

(a) Is the curve $\alpha(t) = \left(\cos t, \cos^2 t, \sin t\right)$ regular ? If so then find the equation of tangent line at $t = \frac{\pi}{3}$.

(b) If $g:[c,d]\to[a,b]$ is a reparametrization of a curve segment $\alpha:[a,b]\to R^3$ then prove that the length of α is equal to the length of $\beta=\alpha\circ g$.

3 Attempt the following questions : **14**

(a) Define the arc length of a curve and prove that the arc length is one - one function mapping *(a, b)* onto *(c, d)* and it is a reparametrization.

(b) Find the arc length of the curve $\alpha(t)=(r\cos t, r\sin t, 0)$ and reparametrize the curve by its arc length.

OR

(b) Show that the length of the curve $\alpha(t)=\left(2a\left(\sin^{-1}t+t\sqrt{1-t^2}\right), 2at^2, 4at\right)$ between the points $t=t_1$ *to* $t=t_2$ is $4a\sqrt{2}\,(t_2-t_1)$.

4 Attempt the following questions : **14**

(a) Show that the curve

$\alpha(S)=\left(\frac{5}{13}\cos S, \frac{8}{13}-\sin S, -\frac{12}{13}\cos S\right)$ is a unit speed curve. Also compute its frenet - Serret appartus.

(b) State and prove Frenet - Serret theorem.

5 Attempt any **two** questions : **14**

(a) Let $f:X\to R^3$ be a simple surface and $f:v\to u$ is a co-ordinate transformation then prove that $y=X_o\ f:V\to R^3$ is also a simple surface.

(b) Let $x:u\to R^3$ be a simple surface then prove that

(i) $x_{ij}=L_{ij}n+\sum_{k}\Gamma_{ij}{}^{k}x_k$

(ii) For any unit speed curve

$\gamma(S)=x\left(\gamma'(S), \gamma^2(S)\right), k_n=\sum_{i,j}L_{ij}\left(\gamma^i\right)'\left(\gamma^i\right)'$ and

$$k_gS=\sum_{k}\left[\left(\gamma^k\right)^n+\sum_{i,j}\Gamma_{ij}{}^{k}\left(\gamma^i\right)'\left(\gamma^j\right)'\right]x_k$$

(c) Define Monge patch and compute coefficients of second fundamental form and Christoffel symbols for the same.