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# PC-003-1163004

Seat No. ______

## M. Sc. (Sem. III) (CBCS) Examination

**June / July - 2018**

## MATHS - CMT - 3004 : Discrete Mathematics

*(New Course)*

**Faculty Code : 003**
**Subject Code : 1163004**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instructions :** (1) Answer all the questions.
(2) Each question carries 14 marks.

**1** Answer any **seven** : **7×2=14**

(a) Define a *homomorphism of semigroups*. Let $f:(S,*)\to(T,*')$ be a surjective homomorphism of semigroups. If $(S,*)$ is commutative, then verify that $(T,*')$ is commutative.

(b) When is a lattice $(L,\leq)$ said to be *distributive* ? Mention an example of a distributive lattice.

(c) Define the concept of *regular expressions* over a set $A$.

(d) Give the details an example of an equivalence relation $R$ on $\mathbb{Z}$ such that $R$ *is* not a congruence relation on $(\mathbb{Z},+)$.

(e) Let $R_1, R_2$ be relations defined on a nonempty set $A$. If $R_i = R_i^{-1}$ for each $i \in \{1, 2\}$, then show that $R_1 \cup R_2$ is symmetric.

(f) Let $G$ be a phrase structure grammar. When is $G$ said to be of type 3 ?

(g) Define : (i) *a finite state machine and* (ii) *a Moore machine.*

(h) Define an *atom* of a bounded lattice. Find all the atoms of $D_{2468}$.

(i) State the *fundamental theorem of homomorphism of semigroups.*

(j) Define : (i) *a conditional statement* and (ii) *a tautology.*

2 Answer any **two** : **2×7=14**

(a) Let $V$ be a nonzero vector space over a field $F$. Prove that the lattice of subspaces of $V$ is distributive if and only if $\dim_F V = 1$.

(b) Let $R$ be a relation defined on a nonempty set $A$. Show that $R^\infty$ is the transitive closure of $R$.

(c) Let $R$ be a congruence relation defined on a semigroup $(S, *)$. Describe in detail the construction of the quotient semigroup of $S$ determined by $R$. Let $A$ be a nonempty set. Show that $(\mathbb{N} \cup \{0\}, +)$ is isomorphic to a quotient semigroup of $A^*$.

3 (a) Let $n \geq 1$. Let $f_1, f_2 : B_n \to B$. Show that **5**

(i) $S(f_1 \vee f_2) = S(f_1) \cup S(f_2)$

(ii) $S(f_1 \wedge f_2) = S(f_1) \cap S(f_2)$

(iii) $S((f_1)') = B_n \setminus S(f_1)$

(b) Let $(L, \leq)$ be a Boolean Algebra. Let $a, b \in L$. Show that (i) $(a \vee b)' = a' \wedge b'$ and (ii) $(a \wedge b)' = a' \vee b'$. **5**

(c) Prove that there exists no semigroup homomorphism from $(\mathbb{N}, \times)$ onto $(5\mathbb{N}, \times)$. **4**

**OR**

3 (a) Let $(L, \leq)$ be a finite Boolean Algebra. Let $a \in L, a \neq 0$. Let $\{c_1, \ldots\ldots, c_k\}$ be the set of all atoms $c$ and $L$ such that $c \leq a$. Prove that $a = V_{i=1}^{k} C_i$. **5**

(b) Let $f : B_4 \to B$ be such that $S(f) = \{0000, 0001, 0011, 0010, 1000, 1001, 1111\}$. Construct the Karnaugh map of $f$ and find a Boolean expression for the function $f$. **5**

(c) Let $R$ be a congruence relation defined on a group $G$. Show that $N = \{g \in G \mid g \, \mathrm{Re}\}$ is a normal subgroup of $G$. **4**

4 Answer any **two** : **2×7=14**

(a) Let $(L, \leq)$ be a lattice. If $(L, \leq)$ is not modular, then prove that $(L, \leq)$ will contain a sublattice $M$ which is isomorphic to the pentagon lattice.

(b) Let $M = (S, I, F, s_0, T)$ be a Moore machine in which $S = \{s_0, s_1\}, I = \{0, 1\}, F = \{f_0, f_1\}$ where $f_0$ = Identity map on $S, f_1 : S \to S$ is given by $f_1(s_0) = s_1$ and $f_1(s_1) = s_0$ , $T = \{s_1\}$. Find $L(M)$. Construct a type 3 grammar $G$ such that $L(M) = L(G)$. Also determine a regular expression $\alpha$ over $I$ such that $L(M)$ is the regular set that corresponds to $\alpha$.

(c) Let $G = (V, S, v_0, \mapsto)$ be a phrase structure grammar in which $V = \{v_0, v_1, a, b\}, S = \{a, b\}$, and the production relation $\mapsto$ is given by $1. v_0 \mapsto av_1, 2. v_1 \mapsto bv_0$ and 3. $v_1 \mapsto a$. Find $L(G)$ and construct a regular expression $\alpha$ over S such that L(G) is the regular set that corresponds to $\alpha$.

5 Answer any **two** : **2×7=14**

(a) Let $P$ be a propositional function. Prove

(i) $\sim (\forall x P(x)) \equiv \exists x \sim P(x)$ and

(ii) $\sim (\exists x\, P(x)) \equiv \forall x (\sim P(x))$

(b) State and prove the Pumping lemma.

(c) Let $R$ be a relation defined on a nonempty finite set $A$. Describe Warshall's Algorithm for computing the transitive closure of $R$.

(d) Let $f : L_1 \to L_2$ be a bijection, where $(L_i, \leq_i)$ is a lattice for each $i \in \{1, 2\}$. Show that the following statements are equivalent.

(i) $f(a_1 \vee a_2) = f(a_1) \vee f(a_2)$ for all $a_1, a_2 \in L_1$.

(ii) For any $a_1, a_2 \in L_1, a_1 \leq_1 a_2$ if and only if $f(a_1) \leq_2 f(a_2)$.