PG-003-001617 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

July - 2018

BSMT - 602 (A) : Mathematics

(Mathematical Analysis - 2 & Group Theory - 2)

Faculty Code : 003

Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Write answer of each question in your main answer sheet.

1 Answer the following questions in briefly : **20**

(1) Define Field

(2) State the first fundamental theorem of homomorphism

(3) Define constant polynomial

(4) Define Homomorphism

(5) Find zero divisor of the ring $(z_6, +_6, \circ_6)$

(6) Define Kernel of homomorphism

(7) If polynomial $g = (0, 2, -3, 0, 0, 0, ...)$ then find degree of g.

(8) Give an example of a ring without unity

(9) Define Ring with Unity

(10) Find characteristic of the ring $(z_6, +_6, \circ_6)$

(11) Define : compact set

(12) Define countable set

(13) Find $L(Sinh\ 2t)$

(14) Check whether the subset $\{2\}$ of metric space R is compact or not

(15) Find $L\left(e^t\ t\right)$

(16) Find $L^{-1}\left(\frac{1}{s-2}\right)$

(17) Show that R is not compact set

(18) Define Connected sets

(19) Determine whether set $\{1,3,5,7,9,11\}$ is connected

(20) Find $L\left(t^{-1/2}\right)$

2 (A) Attempt any **three** : **6**

(1) Prove that every finite subset of any metric space is compact

(2) Find Laplace transform of $\sin^3 2t$

(3) If A and B are compact subsets of metric space R then show that $A \cap B$ is also compact

(4) Show that subset $R - \{2\}$ is not connected

(5) State and prove first shifting property of Laplace Transform

(6) Find Laplace inverse transform of $\frac{s}{(s+a)^2}$

(B) Attempt any **three** : **9**

(1) Show that every singleton subset of any metric space is connected

(2) State and prove Heine-Borel theorem

(3) If F is a closed subset of metric space X and K is a compact subset of X Then prove that $F \cap K$ is also compact

(4) If $L\{f(t)\} = F(s)$ then prove that

$$L\left\{t^n\ f(t)\right\} = -\frac{d^n}{ds^n}\bar{f}(s)$$

(5) Find Laplace transform of $\frac{1-e^{-t}}{t}$

(6) Find inverse Laplace transform of $\cot^{-1}\left(\frac{s}{a}\right)$

(C) Attempt any **two** : **10**

(1) State and prove theorem of nested intervals

(2) Let (X,d) be a metric space and E_1, E_2 are connected sets of X. If $E_1 \cap E_2 \neq \phi$ then prove that $E_1 \cap E_2$ is also connected

(3) Prove that every compact set of a metric space is closed

(4) Find inverse Laplace transform of $\log\frac{s+b}{s+a}$

(5) Find inverse Laplace transform of $\frac{1}{\left(s^2+a^2\right)^2}$ by convolution theorem

3 (A) Attempt any **three** : **6**

(1) Let $\phi:(G,*)\to(G',\Delta)$ is Homomorphism. If $H' \leq G'$ then prove $\phi^{-1}(H) \leq G$

(2) If $\phi:(G,*)\to(G',\Delta)$ is Homomorphism. Then $\phi(e)=e'$ where e & e' are identity elements of $G \,\&\, G'$ respectively.

(3) If $\phi:(G,*)\to(G,*)$, $\phi(x)=x; \forall x \in G$ is homomorp-hism then find K_ϕ

(4) For element a and b of a ring R, prove that a $0=0$

(5) $f(x)=(2,3,4,2,0,0....)$ and $g(x)=(4,2,0,0,3,0....)\in R[x]$ then find $f(x)+g(x)$.

(6) Let I be an ideal of a ring R with unity. Then prove that $\mathrm{I}=\mathrm{R}$ if $1 \in I$

(B) Attempt any **three** : **9**

(1) Prove that a commutative ring with unity is a field if it has no proper ideal

(2) Prove that A Homomorphism $\phi:(G,*)\to(G',\Delta)$ is one-one iff $k_\phi=\{e\}$

(3) Find all homomorphism's of $(Z,+)$ onto $(Z,+)$.

(4) State and prove factor theorem of polynomials

(5) Give an example of a subring which is right ideal but not left ideal.

(6) Prove that any field is an integral domain

(C) Attempt any **two** : **10**

(1) State and prove Remainder theorem

(2) State and prove fundamental theorem of homomorphism

(3) If $f(x) = 4x^4 - 3x^2 + 2$ is divided by $g(x) = x^3 - 2x + 1$ then find quotient $q(x)$ and remainder $r(x)$

(4) If R is a ring and R_1 and R_2 are sub rings of R then show that $R_1 \cap R_2$ is also a sub ring of R.

(5) State and prove division algorithm for polynomials.
