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PF-003-001616

Seat No. ______

B. Sc. (Sem. VI) (CBCS) Examination

July - 2018

Mathematics : 601 (A)

(Graph Theory & Complex Analysis - 2) (New Course)

Faculty Code : 003

Subject Code : 001616

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures on the right side indicate marks.

1 Answer the following questions briefly : **20**

(1) Define : Regular graph.
(2) Write Nullity of connected graph with n vertices and e edges.
(3) Find the number of pendant vertices in any binary tree with 23 vertices.
(4) Define : Trees.
(5) Write relation between internal vertices and pendent vertices.
(6) What is the edge connectivity of a tree.
(7) Define : Seprable graph.
(8) Find the region of a connected planar graph with 4 vertices and 6 edges.
(9) Define : Self dual graphs.
(10) Kuratowski's second graph $K_{3,3}$ has ________ edges.
(11) Write sum function of the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.
(12) Find radius of curvature for the series $\sum_{n=1}^{\infty} n! z^n$
(13) Find fixed points of the mapping $w = \frac{4+5z}{5+z}$.
(14) Find image of circle $|z-1| = 1$ in the complex plane under the mapping $w = u + iv = \frac{1}{z}$
(15) Find critical point of $w = \frac{z-1}{z+1}$

(16) Write formula to find Residue of $f(z)$ at a simple pole $z = a$.

(17) Find Residue of $f(z) = \frac{z+2}{(z-1)(z-2)}$ at $z_o = 2$.

(18) Find singular points of the function $f(z) = \frac{1}{z(z-1)}$.

(19) Find $Res\left(\tan z, \frac{\pi}{2}\right)$.

(20) Which contour is used to integrate $\int_0^\infty \frac{dx}{1+x^2}$.

2 (a) Attempt any **three** : **6**

(1) Define : Simple graph, Isolated Vertex.

(2) What is the minimum hieght of n vertex binary tree ? Write it.

(3) Prove that A graph is a tree if it is minimally connected.

(4) If G is a simple, connected planar graph with of regions, n vertices and e edges $(e > 2)$ then prove that $e \le 3n - 6$.

(5) Define : Path matrix.

(6) Define : Coverings, Minimal coverings.

(b) Attempt any **three** : **9**

(1) State and prove graph theory's first theorem.

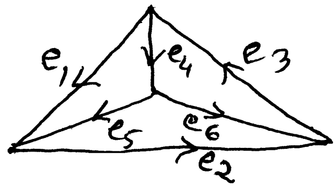
(2) What is the smallest integer n such that the complete graph K_n has at least 500 edges ?

(3) Prove that a tree with n vertices has $n - 1$ edges.

(4) If simple graph G has n vertices, e-edges and f-faces and each face has K-edges then prove that $e = \frac{k(n-2)}{k-2}$.

(5) Define : Minimal dominating set, Domination numbers.

(6) For the following graph G, find minimal decyclization :



(c) Attempt any **two** : **10**

(1) Explain Konigsberg bridge problem and the solution given by Euler.

(2) State and prove necessary and sufficient condition for a graph to be disconnected.

(3) Prove that the complete graph of five vertices is non planar.

(4) State and prove Euler's formula for a connected planar graph.

(5) Define : Incident Matrix and state its properties.

3 (a) Attempt any **three** : **6**

(1) Prove that $\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + ...$

(2) Define : Bilinear mapping.

(3) Show that $x + y = 2$ transform into the Parabola $u^2 = -8(v-2)$ under the transformation $w = z^2$.

(4) Find the residue and pole of order of the function $f(z) = \frac{\sinh z}{z^4}$.

(5) Evaluate $\int_c \frac{5z-2}{z(z-1)} dz$ where, $c : |z| = 2$.

(6) Find $Res\left[\frac{e^{2z}}{(z-1)^2}, 1\right]$.

(b) Attempt any **three** : **9**

(1) Expand $f(z) = \frac{z}{(z-1)(z-3)}$ in Laurent's series for the region $0 < |z-1| < 2$.

(2) Find the image of the infinite strip $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$.

(3) Prove that the transformation $w = 2z + z^2$ maps the unit circle $|z| = 1$ of z-plane into a cardiode in w-plane.

(4) If $f(z)$ has a pole of order 1 at $z = z_0$ then prove that the residue of $f(z)$ at $z = z_0$ is given by $\lim_{z \to z_0} \left[(z - z_0) f(z) \right]$.

(5) Find the value of the integral $\int_c \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$ Taken counter clockwise around the circle $C : |z - 2| = 2$.

(6) Find the linear fractional transformation that maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto the points $w_1 = 1, w_2 = i, w_3 = -1$.

(c) Attempt any **two** : **10**

(1) State and prove Taylor's infinite series for an analytic function.

(2) Show that the circle $|z| = 2$ transform into the ellipse under the transformation $w = \frac{1}{2}\left(z + \frac{1}{z}\right)$ where $z \neq 0$.

(3) Show that composition of two bilinear transformation is again a bilinear transformation.

(4) Prove by using Cauchy residue theorem

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}$$

(5) Prove by using Cauchy residue theorem

$$\int_0^{\infty} \frac{dx}{\left(x^2+1\right)^2} = \frac{\pi}{4}.$$